Strategic Delegation in a Legislative Bargaining Model with Pork and Public Goods

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Abstract

This paper examines the incentives of voters to appoint legislators with different preferences from their own. The paper adopts an underlying legislative bargaining model proposed by Volden and Wiseman (2007) in which legislators with heterogeneous preferences divide a fixed budget between a public good and pork projects (local public goods). We show that voters have an incentive to strategically delegate to affect how the budget is divided at the legislative level. When voters' preferences for pork are not too strong, the incentives for strategic delegation exist to appoint representatives who will direct more money to the public good and not to pork projects. This generally results in at least as many representatives as districts that favor the public good. The comparative statics predict that when strategic delegation occurs, increasing the size of the legislature increases the fraction of the budget spent on the public good.

Keywords: strategic delegation, legislative bargaining, collective decisions, public goods, pork (JEL classification: C78, D71, D72, H4)

1. Introduction

Collective decision-making is a hallmark not only of representative democracies but also city councils, international federations, boards of directors, and academic committees. Frequently, the representatives on these bodies are appointed by others to make policy or large-scale decisions in their place. How are these appointment decisions made? Should we expect representatives to have the same preferences as those that appointed them?

A recent strand of the economics and political science literature has shown that in legislative environments where districts elect representatives to a legislature, the district's choice of representative will depend on its preferences and the underlying legislative bargaining process. This literature shows that sending a representative with different preferences than the district—delegating *strategically*—can advantageously affect how the budget is divided

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and, in particular, can increase the amount of *particularistic goods* or *pork* (local public goods) allocated to the district.

This paper analyzes a modified environment which emphasizes that funds for pork come at the expense of public goods when both are funded from a common budget. Legislators and districts are assumed to have heterogeneous preferences over these goods, and the bargaining process follows a recent model by Volden and Wiseman (2007). Using that model, we ask whether districts prefer to delegate strategically to a representative with different preferences or *sincerely* to a representative with identical preferences. To keep the terminology simple, we refer to legislators and districts that prefer pork as *hogs* and those that prefer the public good as *pubs*.

To illustrate the idea, consider the Lower Manhattan Development Corporation which was assigned the task of distributing a set amount of federal funds after September 11th to rebuild the World Trade Center site. Members of the committee to choose the site designs were appointed separately by the mayor and the governor, occasionally under significant pressure from outside groups. The funds could be spent on features of the project with targeted (particularistic) benefits (e.g., transportation infrastructure, office space, retail space, design, a memorial to the victims) and on speeding up the completion of the project, which was in everyone's interest. Should the competing groups necessarily want a representative on the committee with the same preference?

In this environment we also show that districts will strategically delegate in equilibrium to affect how the budget is divided. But to the best of our knowledge, this is the first paper to show that strategic delegation exists only to increase the funding for the public good and not to attempt to procure more pork for the district. In equilibrium there are generally at least as many pub representatives as districts and, as a result, a weak increase in public goods spending.[1] Because strategic delegation in these cases occurs when public goods spending is efficient, the strategic delegation equilibrium is also welfare improving. Comparative statics show when that when strategic delegation occurs, increasing the size of the legislature increases the fraction of the budget spent on the public good.

The result relies on the underlying bargaining process, however the basic intuition is straightforward. The legislator chosen to divide the budget is unknown at the time voters make their appointment decision. Conditional on a district's legislator being selected to propose a division of the budget, the district always prefers to appoint sincerely in order to maximize contributions to its preferred good. However, conditional on another district's legislator being chosen, it can be advantageous for a hog district to appoint a legislator who

[1] We show that there are equilibria with fewer pub representatives than districts, but counterintuitively, strategic delegation still occurs to direct more funds to the public good. See Section 4 for details.

cares more for the public good. The reason is that more legislators supporting the public good are able to bargain harder for higher contributions to it. If the district is not too likely to receive pork then its payoff comes mostly from the public good anyway, in which case strategic delegation can be optimal when the preference for pork is not too strong. Surprisingly, pub districts never strategically delegate, even when the hogs' preference for pork is strong, hogs are a majority of the legislature, and both types of proposers build winning coalitions using positive amounts of pork.

It is important to note that the results rely on several assumptions. First, the legislature only bargains over funding for a single public good. Funds not spent on the public good go to pork projects, but they can not go to other public goods. Second, there are only two types in the model; we assume districts and legislators either prefer the public good or they prefer pork. Moreover, we assume each hog (pub) has the same relative preference for pork (public good). This means we do not have the opportunity for an "extreme" hog to appoint a more "moderate" hog as legislator. Finally, we assume voters and legislators have linear utility so that the marginal utilities of pork and public good remain constant. Each of these assumptions is done to make the model more tractable, and we discuss the implications in Section 5.

The paper proceeds as follows. We review the related literature in Section 2. The equilibrium results from the legislative bargaining model in Volden and Wiseman (2007) appear in Section 3 along with a discussion of the delegation stage. We present the equilibrium with strategic delegation in Section 4. Comparative statics and welfare results are analyzed. Section 5 summarizes and discusses avenues for further research.

2. Related Literature

There is a long literature on strategic delegation and bargaining. Schelling (1956) notes that players in a bargaining game may be able commit or to bind their hands in order to secure better outcomes. One way to do this is for players to sign contracts with agents for which the payoffs are designed to induce the agent to make choices that limit the bargaining power of the opponent. Fershtman et al. (1991) examines this type of game when contracts are common knowledge and Bester and Sákovics (2001) investigates the effects of allowing contract renegotiation. Another way is to delegate the bargaining to an agent who has inherently different preferences as occurs in two-person games in Jones (1989) and Segendorff (1998).

The same bargaining power incentives exist in the literature on strategic delegation to legislatures, though the literature varies in what agents are trying to accomplish. Besley and Coate (2003) and Dur and Roelfsema (2005) show that agents may strategically delegate to take advantage of the common pool budget problem. In Klumpp (2010) agents use strategic delegation to insure against extreme policy outcomes. In a paper more closely related to this one, Brückner (2000) shows that in a legislature with unanimity rule, strategic delegation to a legislator with preferences closer to the status quo can help a district receive more particularistic goods because that legislator can credibly commit to preferring the status quo outcome.

There are other reasons for a district to strategically delegate. Being a part of the winning coalition at the legislative level can be valuable. This might be the case, for example, when a legislative majority is able to expropriate the minority or allocate the budget to its preferred interests. A district might then strategically delegate to help ensure its legislator is included in the winning coalition. Chari et al. (1997) studies a model in which districts appoint legislators with greater preference for pork because these legislators are the "cheapest" votes for the formateur to obtain. Recently, Harstad (2010) investigates a model in which voters must trade off the incentive to appoint a legislator with preferences close to the status quo with the incentive to appoint one farther away. The former maximizes the district's bargaining power, while the latter increases its chances of being in the winning coalition (it maximizes its "political power"). Harstad shows that the choice between the two turns on the majority voting requirement in the legislature. But either way, the emphasis is on the importance of obtaining particularistic goods (or side-payments) when voters make delegation decisions.

Like Harstad, districts in this paper face a trade-off between bargaining power and political power when appointing a legislator. However, the model in this paper operates in a different environment than models in the previous literature and seeks to answer a different set of questions. We use the Volden and Wiseman model in which funds for pork and public goods come from a common budget.[2] This creates a trade-off for voters in the appointment process between attempting to direct more funds to the public good and more funds to pork projects for their district. Unlike the above papers which contain both public policy and transfers, contributions to pork projects affect the public policy decision and can negatively affect social welfare by diverting funds away from programs with a higher marginal social benefit. We analyze how decisions depend on voters' preferences between the two types of goods, the distribution of those preferences, and the bargaining process in the legislature.

[2]Recently, a handful of papers emphasize that money for pork and the public good come from the same budget. See Leblanc et al. (2000), Lizerri and Persico (2001), and Battaglini and Coate (2008). However, none of these papers considers a legislative setting in which preferences for pork and public goods are heterogeneous, and both types of spending create benefits in the current period.

3. The Model

3.1. Volden & Wiseman's Legislative Bargaining Model

The model underlying this paper is an extension of the baseline model from Volden and Wiseman (2007). Assume $n \geq 3$ legislators (where n is odd) bargain over two types of goods: particularistic goods which give utility only to the legislator to whom they are allocated, and public goods which are enjoyed by all legislators. We assume legislators have either a high, H, or low, L, taste for particularistic spending. Let α_H and α_L be the marginal utilities for the high and low types, respectively, and q the marginal utility for the public good. These marginal utilities are assumed to be constant for all spending levels. To simplify the analysis we normalize the marginal utilities by q and define $k_H = \alpha_H/q$, $k_L = \alpha_L/q$ as the relative preference for pork. We assume $k_L < 1 < k_H$. We refer to high types as hogs and low types as pubs. Hogs are partial to "pork" while pubs are partial to the public good. Throughout the paper we will use high (low) and hog (pub) interchangeably. We also use female pronouns for high types and male pronouns for low types.

Let m be the number of pubs and $n - m$ the number of hogs in the legislature. Utility is assumed to be separable and for legislator i is given as

$$U^i(x^i, y, \theta) = k_\theta x^i + y,$$

where x is the particularistic good, y the public good, and $\theta \in \{H, L\}$ is the legislator's type. We normalize the government's budget to 1 and assume it must spend within its means so that $y + \sum_{i=1}^{n} x^i \leq 1$.

Legislative proposals are made according to a random recognition rule formulated by Baron and Ferejohn (1989). Legislator i is randomly chosen to make a proposal $\Psi = (x^1, x^2, ..., x^n, y)$ that allocates the budget between particularistic projects (a non-negative amount for up to n districts) and the public good subject to budget balance. It is assumed that each legislator has an equal chance of being chosen as the proposer. Ψ is then brought to a vote on the floor where it is compared to a status quo policy whereby funds go unspent and are neither allocated to particularistic nor public goods. If a majority of legislators approve of the proposal it passes and payoffs are awarded.[3] If the proposal does not pass, the game continues to the next period in which a new proposer is randomly chosen.[4] In this case the budget shrinks by the common discount factor, $0 \leq \delta \leq 1$.

The solution concept to the bargaining model is stationary subgame perfect equilibrium (SSPE).[5] To characterize the equilibrium Volden & Wiseman (hereafter VW) divide the

[3] Volden & Wiseman also analyze an open-rule extension to their baseline model.
[4] The same proposer can be chosen in consecutive periods.
[5] See Baron and Ferejohn (1989) for a thorough discussion of stationary equilibria.

parameter space, consisting of ordered pairs (k_H, m), into five regions. For each region the equilibrium proposal for each type of legislator is specified along with a characterization of the winning coalition. In a slight departure from VW's notation, let y^θ_τ be the amount given to the public good by a proposer of type θ, $\hat{x}_\tau$ be the amount given in particularistic goods to the proposer, and x^θ_τ the amount given in particularistic goods to coalition members when the proposer's type is θ.[6] $\tau \in \{A_1, A_{2a}, A_{2b}, A_{2c}, A_{2d}\}$ denotes one of the five divisions of the parameter space specified in the equilibrium proposition and illustrated in Figure 1.

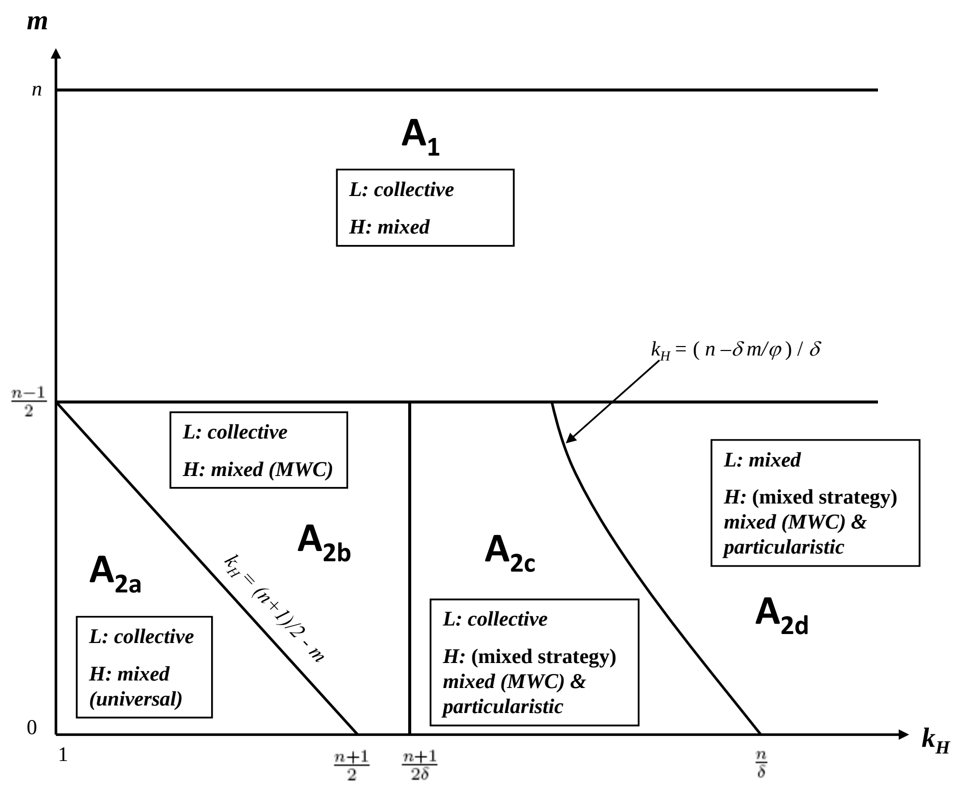
A formal statement of the equilibrium proposition along with the proof can be found in Volden and Wiseman (2007), Volden and Wiseman (2008), and Appendix A. Figure 1, a graphical interpretation of the equilibrium, plots the number of pubs against the intensity of preference for pork spending amongst high types, and identifies the equilibrium spending characterization for the five divisions of the parameter space specified in the proposition.[7] Indicated within each region of the parameter space is how a proposer of each type divides the budget between pork and public goods.

A legislature with a majority of pubs, $m > (n-1)/2$, passes largely public goods-oriented legislation. Pub proposers can easily pass fully-funded public goods bills. A hog proposer is able to take some pork for herself and place the remainder in the public good (a "mixed" proposal), while the amount she can extract in the form of pork is tempered by the ability of the majority to reject any extreme proposal and have a good chance of passing $y^L_1 = 1$ in a subsequent period. Since the proposer's contributions to the public good get all pubs' support, and because pubs form a majority in the legislature, no additional pork spending is necessary to pass the bill.

The characterization of the winning proposal in which the majority of legislators are hogs, $m \leq (n-1)/2$, depends heavily on the intensity of preference for pork for the hog legislators. Consider that there are three types of proposals a hog from district i can make depending on the composition of the legislative body: (1) allocate pork only to herself, and place enough in the public good to get all legislators' approval (a "mixed-universal" proposal), (2) allocate pork to herself, contribute enough to the public good to gain the pubs' support, and include enough additional pork projects to a handful of hogs to fill out a winning coalition (a "mixed minimum-winning coalition" (MWC) proposal), and (3) propose a division that allocates money only in the form of pork projects to a majority of

[6] There is no need to specify the type of representative receiving particularistic goods since in equilibrium pubs will neither propose particularistic goods for themselves nor receive them as part of the coalition since pork spending always goes to those who favor it the most. Moreover, it is inefficient to allocate a pub particularistic goods since they receive a higher marginal utility from the public good, and contributions to the public good also improve the utility of the proposer.

[7] Figure 1 is a reproduction of VW's Figure 3 incorporating the changes from the erratum and from Appendix A.



Notes: $\theta \in \{H, L\}$ denotes the equilibrium proposal by each representative type. For the threshold separating A_{2c} and A_{2d}, $\varphi = \frac{\frac{\alpha_H}{q} - \frac{n+1}{2}}{\frac{\alpha_H}{q} - \frac{n+1}{2\delta}}$.

Figure 1. Equilibrium characterization of the legislative subgame

hogs (a "particularistic" proposal). If k_H is low there is little difference in utility between pork and the public good, and the proposer receives utility from every single dollar spent on either good so a type (1) proposal is attractive. If k_H takes on more moderate values, hogs will no longer support a proposal in which they are allocated only public goods (or such a proposal leaves the proposer too little left over for pork). Instead, it becomes necessary for the hog proposer to offer a type (2) proposal, whereby a coalition of collective members is rounded out with pork projects for a subset of hogs. Finally, for high k_H, offering a particularistic proposal is attractive for a hog proposer and the prospective coalition members. However, this can not be a pure strategy equilibrium as it would guarantee the pubs a low continuation value for the game and give hog proposers an incentive to deviate and offer a mixed–MWC proposal. In equilibrium the hog proposer will mix between a particularistic proposal and a mixed–MWC proposal. These scenarios correspond roughly to areas A_{2a}, A_{2b}, A_{2c}, and A_{2d} in the figure.

A pub proposer facing a majority of hogs benefits from two forces working against the hogs upon the rejection of his proposal: the budget size is reduced by δ, and rejecting a proposal does not ensure a hog that she will be allocated pork in the next period, particularly if the hog majority is large. Hence, the proposer can get a fully funded public goods bill passed even for moderately high k_H. On the other hand, once k_H becomes large these forces are overcome by the overwhelming preference for pork, and the pub proposer responds by "greasing the wheels" with pork spending for enough hog districts to form a majority.

3.2. Bargaining Model with a Delegation Stage

Building on Volden & Wiseman, we ask how the bargaining process affects the type of representative appointed by adding a delegation stage to the model. Consider n single-member districts each of which must simultaneously vote to send one representative to the legislature. Representatives run as either a hog or a pub and we assume their true types are known with certainty. Legislators have the same linear utility as median voters.

To simplify the choice problem facing the district, assume that either all citizens in the district have identical preferences (i.e., all are of type H or of type L) or that there is a single pivotal voter whose choice determines the district's representation. Either assumption implies each district's median voter can be characterized as preferring pork for the district or the public good. Furthermore, in order to make the welfare analysis in subsequent sections less cumbersome, assume that each district has an identical number of voters (as in a congressional district). Suppose there is no uncertainty over median voter types.

Following the notation of the VW model, let m denote the number of realized pub legislators and $n - m$ the number of realized hog legislators. To answer questions of delegation, we need notation for the number of pub and hog districts. Let ω denote the number of L districts and $n - \omega$ the number of H districts. The objective then is to understand the relationship between ω and m.

Equilibrium in the delegation game requires that in the legislative subgame legislators choose a coalition and contribution amounts so as to maximize their utility given their preferences over the two types of goods. As in Section 3.1, we limit strategies to those that are history independent. Equilibrium also requires that forward-looking median voters best respond to other district medians by appointing a hog or pub to maximize their expected value in the legislative subgame. Thus the notion of equilibrium is that of subgame perfect Nash equilibrium limited to stationary strategies in the legislative subgame.

4. Strategic Delegation Equilibrium

To solve the delegation game, we take as given the number of pub districts, the marginal utilities, the discount factor, and the total number of districts, and we solve for the equilibrium value(s) of m. For ease in presentation we present the equilibrium value of m separately for a majority of hog districts and a majority of pub districts. Note that there are occasionally multiple equilibria in the model. This is a consequence of the legislative subgame equilibrium in which the nature of the winning coalition changes by region in the parameter space. Since expected payoffs for a given median voter type change discontinuously across regions, it is possible to have a local payoff maximum at the boundary where no single voter has an incentive to deviate from the given appointment choice.

4.1. A Majority of Hog Districts

Suppose hog districts are in the majority, $\omega \leq (n-1)/2$. The equilibrium proposition below derives the relationship between m and ω for low, moderate, and high k_H. The proof appears in Appendix B.

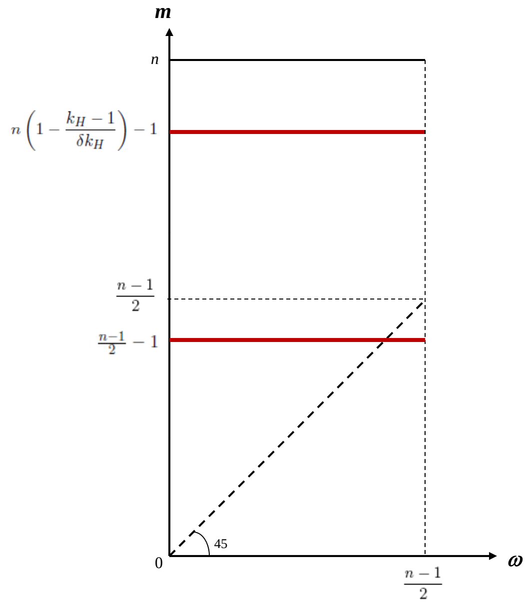
Proposition 1. *Suppose $\omega \leq \frac{n-1}{2}$. The delegation game has the following equilibria.*

1. (low) *Suppose $k_H \leq \frac{n}{n-\delta\left(\frac{n-1}{2}\right)}$. Then,*
   (a) $m^* = \max\left\{n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-1,\ \omega\right\}$. *L districts appoint sincerely and $(m^*-\omega)$ H districts appoint strategically. $m^* \geq \omega$.*
   (b) $m^* = \frac{n-1}{2}-1$. *If $\omega \leq \frac{n-1}{2}-1$ then L districts appoint sincerely and $(m^*-\omega)$ H districts appoint strategically. Otherwise, at least one L district appoints strategically.*[8] $m^* \gtreqless \omega$.
2. (moderate) *Suppose $\frac{n}{n-\delta\left(\frac{n-1}{2}\right)} < k_H \leq \frac{n+1}{2\delta}$. Then,*
   (a) $m^* = \max\left\{n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-k_H,\ \omega\right\}$ *if $\omega \leq \dfrac{n+1}{2}-k_H$. L districts appoint sincerely and $(m^*-\omega)$ H districts appoint strategically. $m^* \geq \omega$.*
   (b) $m^* = \max\left\{n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-1,\ \omega\right\}$. *L districts appoint sincerely and $(m^*-\omega)$ H districts appoint strategically. $m^* \geq \omega$.*
   (c) $m^* = \frac{n+1}{2}-k_H$ *if $\omega \geq \frac{n+1}{2}-k_H$. H districts appoint sincerely and $(\omega-m^*)$ L districts appoint strategically. $m^* \leq \omega$.*
3. (high) *Suppose $k_H > \frac{n+1}{2\delta}$. Then, $m^* = \max\left\{n\left(1-\dfrac{k_H-1}{\delta k_H}\left(\dfrac{k_H}{\frac{n+1}{2\delta}}\right)\right)-1,\ \omega\right\}$. L districts appoint sincerely. $(m^*-\omega)$ H districts appoint strategically. $m^* \geq \omega$.*

Proposition 1 shows that strategic effects can alter the types of representative a district wishes to appoint.[9] The model predicts that there are generally more pub legislators than

[8] This also requires $k_L > \frac{k_H\delta(n+1)/2-n}{\delta(n+1)/2-n}$.

[9] One issue that the equilibrium in Proposition 1 abstracts from is which districts should play strategically when equilibrium calls for more than one (but not everyone) to do so. One way to address this is to allow districts to play mixed strategies, in which the probabilities are straightforwardly determined by the number of districts required to play strategically in equilibrium.



Notes: The figure plots the equilibrium number of pub legislators as a function of pub districts.

Figure 2. Strategic delegation equilibrium for low k_H

pub districts with the exception of the equilibria in 1(b) and 2(c). However, these equilibria only exist for large enough ω indicating that for large hog majorities, there are always more pub legislators than districts. In the equilibria for which there are more pub legislators than districts, as the marginal utility for pork increases for hog districts—and thus as $(k_H - 1)/k_H$ increases—the equilibrium number of pub legislators is weakly decreasing for each range of k_H.[10] In the equilibria for which there are fewer pub legislators, the equilibrium is driven by behavior at the boundaries of the bargaining regions from the VW paper, and in both 1(b) and 2(c) an equilibrium also exists with more pub legislators than districts. We will return to parts 1(b) and 2(c) in section 4.2 when discussing a majority of pub districts since the equilibria appear there as well.

In part 1 of the proposition k_H is low enough that the hogs' marginal utility for pork is less than twice their marginal utility for the public good. The equilibrium number of pub legislators in 1(a) is at least as great as the number of pub districts and, frequently,

[10]Because $k_H, k_L \in \mathbb{R}^+$ the equilibrium values of m as written in Proposition 1 are not restricted to the set of natural numbers. While this is done to economize on notation, it has no real bearing on the results. To be precise, if the equilibrium calls for an increase in the number of pub legislators and the equilibrium $m \notin \mathbb{N}$ then we take the closest natural number greater than m, $\lceil m \rceil$. Conversely, if the equilibrium calls for a decrease in the number of pub legislators then we take the closest natural number less than m, $\lfloor m \rfloor$.

much larger. In fact, for this range of k_H, m^* is greater than $(n-1)/2$. This implies that pubs comprise a majority of all legislators regardless of the number of pub districts. Figure 2 presents the equilibrium number of pub legislators as a function of the number of pub districts for fixed k_H in this range. Notice that as k_H increases m^* will decrease.

It is not in the pub districts' interest to appoint hogs no matter the appointment decisions of the other districts in the equilibrium in part 1(a). For any k_H in this interval there are three possible characterizations of equilibrium in the bargaining game which depend on how many pubs are sent to the legislature. The possible contingencies are A_1, A_{2a}, and A_{2b}, and the bargaining outcome in each has a pub proposer passing $y^L = 1$ and a hog proposer keeping some pork and contributing the rest of the budget to the public good.[11] The pub district prefers a pub representative for each contingency. Not only does appointing a pub make $y^L = 1$ more likely, but supposing the district did appoint a hog and the hog was chosen proposer, the district would rather see its pork project go to the public good!

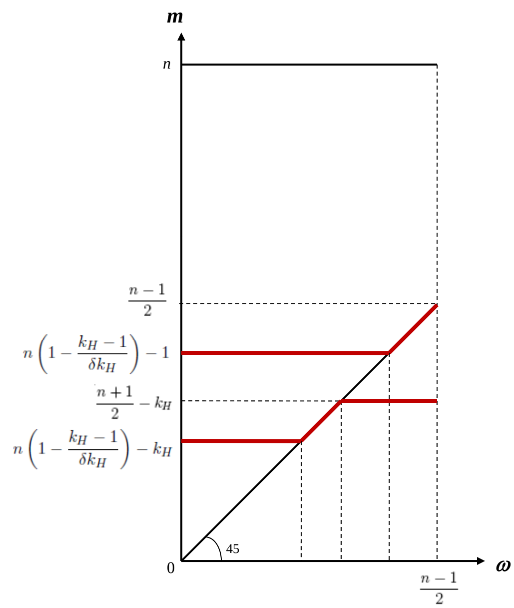
Hog districts, on the other hand, have an incentive to strategically delegate in 1(a). This is because there is relatively little difference between the two types of goods for hog districts and the share of the budget allocated to pork is relatively small. To be more precise, because pork only goes to the bill's proposer when a hog proposes, and given that having one's legislator selected to propose happens with relatively low probability, a hog district often does better by appointing a pub and increasing the bargaining power of the pub legislators: no bill goes through without the pubs' support, and the more likely it is that a pub will propose in the next period, the more the pub legislators can extract for the public good when a hog is the chosen proposer.[12] This way hog districts make it more likely $y^L = 1$ will be offered and they are better off each time another district's hog proposer divides the budget.

One final note about part 1(a) of the proposition is that the number of pub representatives stops short of $m^* = n$. Consider the extreme case in which all districts except one appoint pub legislators, $m = n-1$. A hog district would never wish to delegate strategically in this case. Conditional on another district's representative (a pub) being chosen to divide the budget, she is indifferent, and conditional on her representative being chosen, she wishes to appoint sincerely to get some pork.

Parts 2(a) and 2(b) of the proposition show that for more moderate values of k_H there are again equilibria with at least as many pub legislators as districts. But for this range

[11] In this range for k_H, the only possible outcome in A_{2b} is for $m = (n-1)/2$. A proposal by a hog in this case is essentially a mixed proposal since no other pork projects need to be handed out.

[12] In other words, appointing a pub raises all the pubs' continuation value for the game. It is easy to check that both $\partial y_{2a}^H/\partial m$ and $\partial y_1^H/\partial m > 0$.



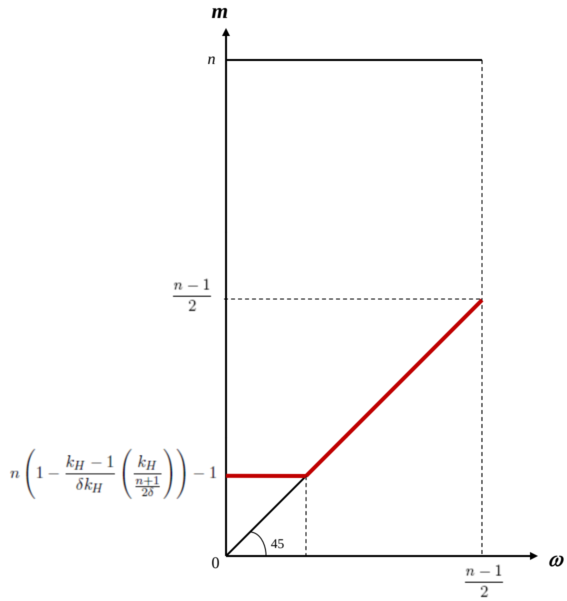
Notes: The figure plots the equilibrium number of pub legislators as a function of pub districts. The figure assumes $n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-1>\left(\frac{n+1}{2}\right)-k_H$ and $n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-k_H>0$.

Figure 3. Strategic delegation equilibrium for moderate k_H

of k_H, it is the case that $m^* \neq \omega$ implies $m^* \leq (n-1)/2$.[13] This is to say, a majority of pub legislators will only occur for moderate k_H if a majority of districts favor the public good. Nevertheless, strategic delegation still has consequences. If ω is low enough there will again be more pub legislators than districts. Figure 3 graphs the equilibrium number of pub legislators as a function of the number of pub districts. Note that increasing k_H in this range will also weakly reduce the equilibrium number of pub legislators regardless of which equilibrium is played.

Once more only hog districts behave strategically in parts 2(a) and 2(b). To understand why, consider the incentives facing the pub districts. For any k_H in this interval there are three possible characterizations of equilibrium in the bargaining game which depend on how many pubs are sent to the legislature. In two of the three, A_1 and A_{2a}, it is clear that a pub district should never delegate to a hog for reasons discussed above. In A_{2b}, however, a hog proposer allocates a handful of pork projects in addition to contributions to the public good, which guarantees those districts with a hog representative that also receive a pork project a higher payoff than the remaining districts ($\theta x_{2b}^H + y_{2b}^H > y_{2b}^H$). Yet strategic delegation

[13]This is easy to see by looking at the effect of the $(k_H-1)/k_H$ term on m^*.



Notes: The figure plots the equilibrium number of pub legislators as a function of pub districts. The figure assumes $n\left(1-\frac{k_H-1}{\delta k_H}\left(\frac{k_H}{(n+1)/(2\delta)}\right)\right)-1>0$.

Figure 4. Strategic delegation equilibria for high k_H

remains unattractive for a pub district. The intuition is that *every time* a hog is chosen to propose and the pub district delegates sincerely, the hog proposer is forced to give a larger share to the public good. If the pub district instead delegates strategically, it only reaps the benefits when it is lucky enough to be in the coalition. Even when this does occur, pork has a small enough value for the pub district that it does not compensate for the opportunity cost of not having a pub legislator.

Given that pub districts never have an incentive to delegate strategically in 2(a) and 2(b), there will never be fewer pub legislators than districts. The hog districts' problem is similar. Because k_H takes on larger values than in part 1 of the proposition it is no longer feasible to have significant amounts of strategic delegation by hog districts. Instead, strategic delegation only occurs when there are few pub districts. In this case, if all hogs were to appoint sincerely, it is not only unlikely that a district's representative would be chosen proposer, but it is unlikely that if they were to send a hog that this legislator would be included in a mixed-MWC (should this type of proposal be optimal in equilibrium). The hog districts then have an incentive to strategically delegate as before to divert more money to the public good and to make it more likely a pub is chosen proposer.

Part 3 shows that in equilibrium there are always at least as many pub legislators as districts when k_H is high. Notwithstanding this result, a small amount of algebra shows that this is a low threshold (when it is positive) because of the range of k_H. Figure 4 graphs the number of pub legislators as a function of pub districts.

The most surprising implication of part 3 regards the behavior of the pub districts. Consider that only two bargaining outcomes are possible for fixed k_H in this range depending on the choices of the other districts. It is possible that a majority of pubs are elected in which case there is no incentive for a pub district to delegate to anyone but a pub. On the other hand, if a majority of hogs are elected then with a certain probability the hogs will make a particularistic proposal in which the budget is divided only into pork projects for other hog districts. A pub district with a pub legislator receives zero payoff from this type of proposal. It appears then that this would provide the impetus for strategic delegation by pub districts to get something, even if it is a less preferred good.

The trade-offs for pub districts become clear in the limit as k_H becomes large. The larger is k_H the more hogs allocate to pork, so if pubs will strategically delegate it will happen here.

Remark 1. *For $k_H > \frac{n+1}{2\delta}$, the expected contribution to the public good is*

$$\left(\frac{m}{n}\right) y_{2d}^L + \left(\frac{n-m}{n}\right)(1-\beta) y_{2d}^H = \frac{\frac{m}{n}}{1 - \left(\frac{1}{k_H}\right)\left(\frac{n-m}{n}\right)\left(\frac{n+1}{2}\right)}.$$

This converges to m/n (the proportion of pubs) from above as k_H goes to infinity. The expected contribution to pork converges to $(n-m)/n$ (the proportion of hogs) from below.

The expected total contribution to the public good converges on the proportion of pub legislators in the limit. Since the expected contribution to the public good is the pubs' expected payoff, the marginal benefit for a pub district of appointing sincerely is $1/n$. The marginal benefit of a hog legislator is the utility from the district's expected share of pork, which approaches $\left(\frac{1}{n-m}\right)\left(\frac{n-m}{n}\right) = \frac{1}{n}$ in the limit. For a pub district the marginal benefit from strategic delegation is k_L/n and since $k_L < 1$ this implies strategic delegation will never be optimal for pub districts.

The intuition is also easiest to understand in this extreme case. Suppose the hogs have a small majority in the legislature. If a pub district appoints a hog, the district stands an excellent shot at receiving pork when a hog proposes since virtually every hog legislator is needed to form a particularistic coalition. However, the expected share of the budget going to pork is relatively small (just over 50%) so that the share for the district is relatively small. Now suppose hogs have a large majority so the expected share of the budget going to pork is large. There is more money going to pork, but that money gets spread out over many more hog districts. It turns out that either way, the expected share of pork a district will get by appointing a hog is always $1/n$ in the extreme case (and less for lower values of k_H). The marginal benefits are k_H/n and k_L/n for hog and pub districts, respectively, and given that the opportunity cost is a reduction of $1/n$ in contributions to the public good (more for lower values of k_H), this is a trade-off pubs are never willing to make. This result only holds because of the fixed budget and the fact that funds for pork and public good are linked. Incorporating an exogenous budget for pork as is generally done in the literature would make pubs quick to strategically delegate (though the model in that case is no longer interesting).[14]

Since pork is extremely valuable to hog districts in this interval, they are willing to strategically delegate only when the chances of receiving pork are small. This occurs when there are very few pubs in the legislature. Because pub districts do not strategically delegate, few pub legislators implies few pub districts. Hence, ω must be quite small for hog districts to delegate strategically.

An alternative way to view Proposition 1 is to graph the relationship between the equilibrium number of pub legislators and the hogs' relative preference for pork given ω. Figure 5 does this for $\omega = 0$ using equilibria 1(a) and 2(a) from Proposition 1 (the cases not driven by behavior on the boundaries). Even when all districts prefer pork, a majority of legislators are pubs for low k_H. For more moderate k_H the majority of legislators are hogs, though pubs can still be a sizeable portion of the legislature. As k_H continues to increase the number of pubs continues to decrease, but notice that the way the figure is drawn, strategic delegation is possible even for very high k_H.[15]

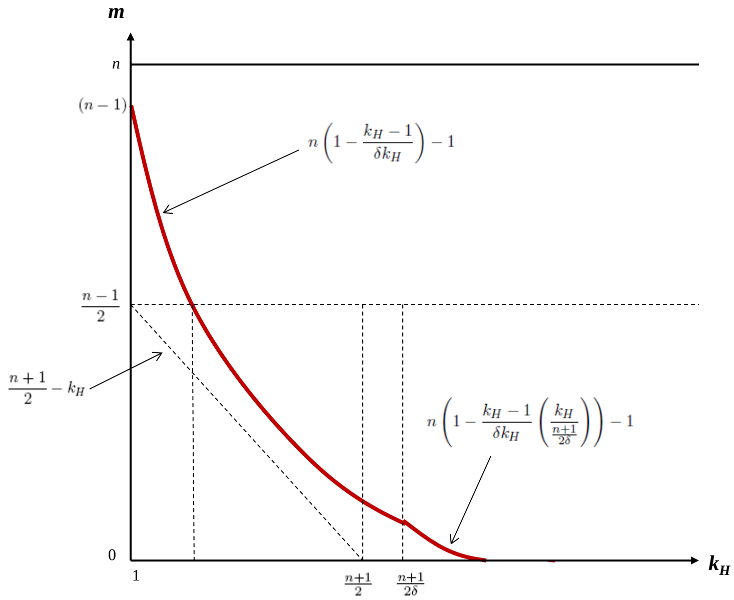
It is worth noting what is *not* driving the strategic decisions of the hog districts. There is a positive externality involved in producing the public good; contributing \$1 yields 1 unit of added utility to *every* player. Hence, if a hog districts delegates to a pub, thereby making production of the public good more likely, this benefits everyone. However, these districts ignore this external benefit; strategic incentives come from purely selfish utility maximization.

4.2. A Majority of Pub Districts

Suppose now that pub districts have a majority, $\omega > (n-1)/2$. The proposition solves for the equilibrium number of pub legislators, m^*, using the same intervals for k_H that appear in Proposition 1.

[14] It also depends on the linear utility assumption; with diminishing marginal utility a pub might be willing to forgo some public good for a chance at pork.

[15] $m^* = \omega = 0$ is only guaranteed for $k_H \geq n$.



k_H	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
m^*	8	7	6	4	3	3	2	1	1	0

Notes: The graph assumes $n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-1 > \left(\frac{n+1}{2}\right)-k_H$. We assume equilibria are characterized by 1(a) and 2(a) from Proposition 1. The numerical results are for $n=11$ and $\delta=0.9$.

Figure 5. Equilibrium m^* for $\omega = 0$

Proposition 2. *Suppose* $\omega > \frac{n-1}{2}$. *The delegation game has the following equilibria.*

1. (low) *Suppose* $k_H \leq \frac{n}{n-\delta\left(\frac{n-1}{2}\right)}$. *Then,*
   (a) $m^* = \max\left\{n\left(1-\dfrac{k_H-1}{\delta k_H}\right)-1,\ \omega\right\}$. *L districts appoint sincerely and* $(m^*-\omega)$ *H districts appoint strategically.* $m^* \geq \omega$.
   (b) $m^* = \frac{n-1}{2}-1$. *At least two L districts appoint strategically.* $m^* < \omega$.
2. (moderate) *Suppose* $\frac{n}{n-\delta\left(\frac{n-1}{2}\right)} < k_H \leq \frac{n+1}{2\delta}$. *Then,*
   (a) $m^* = \omega$. *All districts appoint sincerely.*
   (b) $m^* = \frac{n+1}{2} - k_H$. *H districts appoint sincerely and* $(\omega - m^*)$ *L districts appoint strategically.* $m^* < \omega$.
3. (high) *Suppose* $k_H > \frac{n+1}{2\delta}$. *Then,* $m^* = \omega$. *All districts appoint sincerely.*

For low k_H the equilibrium in 1(a) is the same as exists in Proposition 1. In this case some hog districts strategically delegate to pub representatives because their payoffs are almost always determined by what is contributed to the public good. As before, there are more pub legislators than districts. If k_H takes on moderate or high values, it will be an equilibrium for both sides to appoint sincerely as show in parts 2(a) and 3. By strategically delegating to a hog, a pub district only decreases expected funding to the public good, while hog districts care too much for pork to give up their chance to obtain it should their legislator be chosen as proposer.

The other equilibria of interest in Proposition 2, which also appear in Proposition 1, are given in parts 1(b) and 2(b). These equilibria exist along the boundary of regions A_{2a} and A_{2b}. They result in strategic delegation by pub districts so that the legislature has fewer pub legislators than districts. Even so, as we explain below, the incentives to strategically delegate still stem from the desire to channel more money into the public good. These equilibria along the boundaries are more "fragile" in the sense that any small amount of coordination between districts would negate them, as would enough uncertainty over other districts' types.

Part 1(b), in which hog legislators are in the majority, $m = (n-1)/2-1$, is an equilibrium in this interval because it is a local expected utility maximum. Both hog and pub voters would prefer to see more pubs appointed, but no single voter with the ability to switch his vote and appoint a pub is willing to do so. This is because it results in $m = (n-1)/2$, an outcome in A_{2b}, with a lower expected contribution to the public good. If two or more districts appointing hogs could coordinate to appoint pubs instead, it would be in their interest to do so.

The situation is the same in part 2(b). Suppose that for a given vector of appointment choices $(k_H, m) \in A_{2a}$ and $(k_H, m+1) \in A_{2b}$. Consider a pub district appointing strategically. This district has no incentive to deviate and delegate to a pub representative in order to shift the bargaining outcome to A_{2b}. The reason is that the amount contributed to the public good by a hog proposer when there are m pub legislators and the bargaining outcome is in A_{2a} is actually greater than the amount when there are $m+1$ and the outcome is in region A_{2b}.[16] Since a hog proposer gets unanimous support in A_{2a} using the public good, the pub legislators receive more than their continuation value for the game to support the proposal. Of course, by delegating strategically the pub district decreases the chances of a fully funded public goods proposal, but for large enough k_H, the benefits of receiving y^H_{2a}, as well as possibly a pork project, trump this effect. In equilibrium strategic delegation still occurs to get more public good, not pork.

[16]That is, $y^H_{2a}(m) > y^H_{2b}(m+1)$.

4.3. Social Welfare

When strategic delegation results in more pub legislators than districts it has positive implications for social welfare. This happens for two reasons. Hog districts that delegate to pubs maximize their own utility, and these decisions exhibit positive externalities vis-à-vis contributions to the public good. That is to say, each time a hog divides the budget and allocates something to the public good the proposer allocates more than would otherwise be the case. Strategic delegation also increases the likelihood of a fully funded public goods bill to be enjoyed by the other districts.

We formalize this intuition by establishing the efficient level of expected social surplus and comparing this to the expected surplus under both strategic delegation and sincere appointment for the intervals of k_H in Propositions 1 and 2. Expected social surplus is used since ex ante it is uncertain who will be selected to propose a division of the budget.

Given n single-member districts, m pub legislators, and discount factor δ, the expected social surplus is the sum of expected utilities across voters in each district. So long as voters in each district are identical or the pivotal median voter takes on the average characteristics of the district, we can analyze social welfare using the expected utility of the n median voters. Let $\mathbb{E}[\mathsf{S}_\tau]$ denote expected social welfare in parameter region A_τ where $\tau \in \{1, 2a, 2b, 2c, 2d\}$. Let $d^i \in \{H, L\}$ be the chosen delegate type by district i's median voter (who himself has type $\theta^i \in \{H, L\}$). Then,

$$\mathbb{E}[\mathsf{S}_\tau] = n + \left(\frac{m}{n}\right)(1 - y_\tau^L)\left(\frac{\sum\limits_{i:d^i=H} k_{\theta^i}}{(n-m)} - n\right) + \left(\frac{n-m}{n}\right)(1 - y_\tau^H)\left(\frac{\sum\limits_{i:d^i=H} k_{\theta^i}}{(n-m)} - n\right). \quad (1)$$

If both types of legislator fully fund the public good then the expected social surplus is equal to n. If the entire budget is not devoted to the public good, social surplus can be greater or less than n depending on the average intensity of preference for pork among districts represented by hogs, $\sum\limits_{i:d^i=H} k_{\theta^i}/(n-m)$, the total amount contributed to pork by each type of legislator, and the probability a pub or hog is selected as proposer. The linear utility functions guarantee an easy solution to the social surplus maximization problem.

Lemma 1. *Given a vector of district median voter types, $(\theta^1, \theta^2, ..., \theta^n)$, the efficient budget allocation is $y = 1$ for $k_H < n$ and $\sum\limits_{i:\theta^i=H} x^i = 1$ for $k_H > n$. Any distribution of benefits is efficient for $k_H = n$, provided pork is allocated only to hog districts.*

Note that every dollar put towards the public good yields a utility of 1 to each district and a social surplus of n. Each dollar put towards pork yields only k_H or k_L in social surplus because of the targeted nature of particularistic benefits. While any convex combination of pork and public goods is possible, social surplus is maximized by contributing solely to the one with the higher marginal social value. The efficient outcome will then be either to fully fund the public good or to divide the budget among the districts with $\theta = H$. Only if $k_H = n$ is any allocation of public goods and pork to hog districts efficient.

Define a political equilibrium as the equilibrium outcome of the legislative bargaining game with a delegation stage.

Proposition 3. *Political equilibria with strategic delegation in which $m^* > \omega$ are Pareto optimal, while those with $m^* < \omega$ are not.*

Note that an outcome is only Pareto optimal in this environment if pub districts do not receive pork in equilibrium. If pub districts did strategically delegate and receive pork, redistributing those funds to the public good would be a Pareto improvement.

Proposition 4. *Political equilibria with strategic delegation are not expected social surplus maximizing. However, when*

1. *$m^* > \omega$ expected social surplus is higher with strategic delegation than with sincere delegation.*
2. *$m^* < \omega$ and*
   - (a) *k_H is low, expected social surplus with strategic delegation is higher than sincere delegation when $\omega = (n-1)/2$.*
   - (b) *k_H is moderate, expected social surplus with strategic delegation is higher than sincere delegation if ω, k_H, and k_L satisfy,*

$$k_L > \frac{n^2\left(\omega - \left(\frac{n+1}{2} - k_H\right)\right) - n^2 k_H \delta + k_H \delta\omega\left(\omega - \frac{n+1}{2}\right) + \frac{k_H n\delta}{2}}{\left(\omega - \left(\frac{n+1}{2} - k_H\right)\right)\left(n(1-\delta) + \delta\omega\right)}. \qquad (2)$$

Even with strategic incentives, the outcome of the bargaining process will be inefficient, unless $k_H < n$ and $\omega = n$, $k_H > n$ and $\omega = 0$, or $k_H = n$. For $k_H < n$ recall that in any parameter region if $\omega \neq n$ then with some positive probability a portion of the budget will spent on pork which is inefficient. The size of the inefficiency will depend on the difference between the marginal benefits of a dollar to pork or the public good, the probability with which pork is allocated in equilibrium, and the actual amount of pork allocated. Propositions 1 and 2 show that for $k_H < n$, there are equilibria in which districts that prefer pork will send pubs to the legislature. This delegation strategy not only reduces the chance pork will be allocated in the equilibrium subgame, but it increases by way of bargaining power the amount given to the public good. Because public spending has a higher marginal impact on welfare, this brings the delegation outcome closer to full efficiency. For $k_H \geq n$ no strategic delegation occurs and there is no change in social surplus.

When strategic delegation results in fewer pub legislators than districts, the welfare results are more ambiguous. When k_H is low social welfare increases if $\omega = (n-1)/2$ since one less pub shifts the bargaining outcome to A_{2a} where more money is given to the public good. In all other cases, pubs are in the majority under sincere delegation so moving to a pub minority decreases welfare. For moderate k_H the strategic delegation equilibrium has the potential to be of higher or lower welfare than the outcome under sincere delegation. Intuitively, if a small number of pubs delegate strategically and the outcome shifts to A_{2a} where public goods contributions are higher, this raises welfare.[17] However, if many pubs strategically delegate in equilibrium and the bargaining outcome moves from A_1 to A_{2a} welfare will decrease. The differences are illustrated in Figure 6.[18]

4.4. Comparative Statics

Next we turn to how the equilibrium predictions of the strategic delegation model are affected by its parameters. Comparative statics are done for each equilibrium m^* from Propositions 1 and 2. Comparative statics done across ranges of k_H and ω yield similar predictions but are complicated by discontinuities in m^*.

Proposition 5. *The equilibrium proportion of pub representatives, m^*/n, is*

(a) weakly increasing in δ, and

(b) weakly decreasing in k_H,

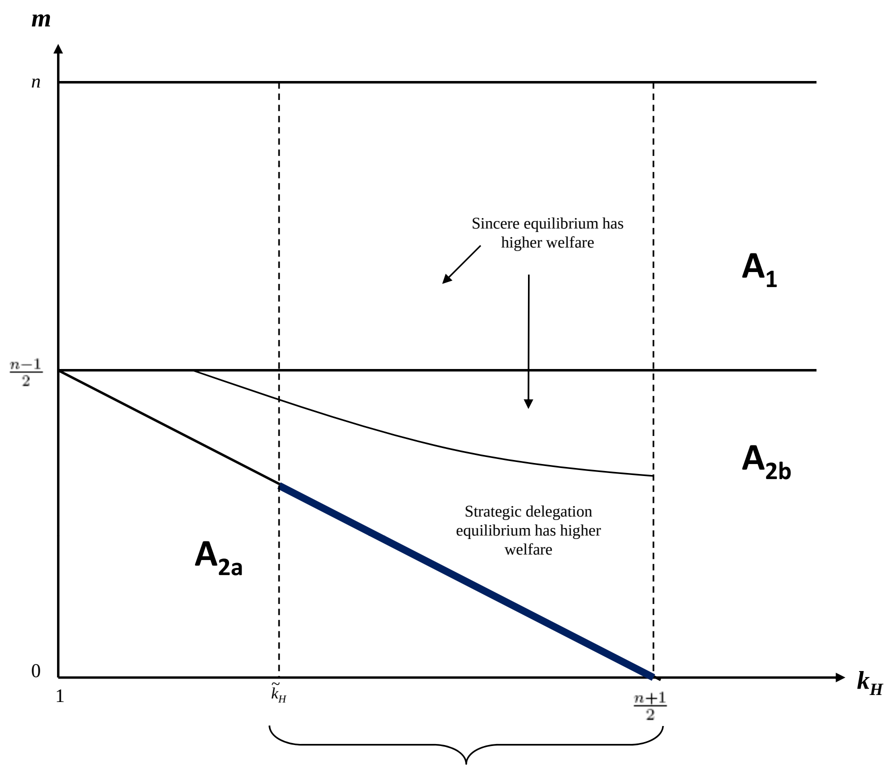
for each specification of m^* in Propositions 1 and 2.

Increases in δ imply greater patience among the representatives in the legislature and among the voters in the districts. Less pressure to get deals done now serves to increase the bargaining power of the representatives needed to form a winning coalition who are now more willing to reject a proposal and bargain again in the next period. This decreases the power of the proposer, and in particular, the power of hogs attempting to take large shares of pork for their district, while increasing funding for the public good. Moderating the proposer's take in pork makes having a hog representative less attractive and serves to increase the amount of strategic delegation by hog districts.[19]

[17] Even though the strategic delegation equilibrium *increases* the probability of a hog proposer, it can compensate by increasing y. Notice that $\partial y_{2a}^H/\partial k_H > 0$ also implies that as k_H increases in this interval there is room for more L districts to appoint strategically and still improve welfare. Formally, let $\overline{\mu}$ denote the maximum number of strategic appointments by L districts such that overall welfare increases. Then $\partial\overline{\mu}/\partial k_H > 0$.

[18] The calculation of expected social surplus behind part 2 of Proposition 4 and Figure 6 is available by request.

[19] The equilibria in which pub districts strategically delegate are not affected by δ.



Notes: $\tilde{k}_H = \frac{n}{n-\delta\left(\frac{n-1}{2}\right)}$.

Figure 6. Welfare comparisons of the sincere and strategic equilibria for moderate k_H

As the intensity of preference for pork among hog districts takes on larger values the equilibrium number of pub legislators decreases. In the equilibria in which hog districts appoint strategically, higher k_H raises the opportunity cost of strategic delegation and decreases the number of pubs appointed. In the equilibrium on the boundary of A_{2a} and A_{2b} in which pub districts appoint strategically, increases in k_H reduce the boundary value of m. This means that any equilibrium on the boundary in which pubs delegate strategically will have fewer pubs when k_H takes on larger values.

Propositions 1, 2, and 5 have potentially testable implications. If we assume voters appoint sincerely then a shock to the relative preference for pork which does not actually change the preference ranking for the districts will only affect the bargaining outcome of the legislative subgame, and it will have no bearing on the number of hogs and pubs in the legislature.[20] Such a shock could be, for example, a change in a country's preference for

[20]By holding the preference ranking constant we mean the marginal utility for one good remains higher

spending at the centralized versus decentralized levels. According to the strategic delegation model, however, the composition of the legislature may also change as a result of such a shock. So while the VW bargaining model would predict an increase in funding for the public good if there is a greater interest in centralization among voters, the prediction might fall short of the actual change in funding if the composition of the legislature also changes.

Proposition 6. *Suppose $m^* > \omega$ for given values of ω, k_H, and δ, and the budget is fixed. The equilibrium proportion of pub representatives, m^*/n, is increasing in n for each specification of m^* in Propositions 1 and 2.*

The reason for this result is that more districts decrease the expected payoff from pork for a hog. For example, suppose there are a nonzero number of hogs and pubs in the legislature, and we double the legislature's size while keeping the proportion of pubs fixed. In the bargaining equilibrium at the legislative level, there are no changes in the total amounts spent on pork and public goods. However, because there are now more hogs in the legislature and the budget is unchanged, each hog's expected share of pork has decreased. This makes hogs more likely to delegate to pubs.

Proposition 6 sheds light on the relationship between the size of the legislature and efficiency. A well known strand of research shows that legislatures with more members may be associated with higher levels of total spending, a phenomenon often referred to as the Law of $1/n$.[21] The idea is that legislators have an incentive to increase local public goods spending (pork) above efficient levels since their district only pays a fraction of the cost of a project. The VW model uses a fixed budget so we are unable to comment on total spending levels. What we can do, however, is comment on how a fixed budget is divided between pork and public goods as the size of the legislature changes. Proposition 6 shows that when strategic delegation is sending more pubs to the legislature, increasing the size of the legislature increases the equilibrium proportion of pub representatives.[22] This means that a larger legislature spends a larger fraction on public goods, holding everything else, including the size of the budget, constant. Since strategic delegation occurs when it is efficient to give to the public good, efficiency increases.

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than that for the other good for all districts. That is to say, hog districts remain hogs and pub districts remain pubs.

[21] See Weingast et al. (1981) and recent papers by Primo and Jr. (2008) and Knight (2006) for discussion.

[22] This result does not necessarily hold if strategic delegation is not already taking place. If $\omega = m^*$ then making the legislature larger could increase or decrease the amount spent on the public good. For example, suppose all of the districts are pubs ($\omega = n$) and the new districts added are hogs. The share of the budget going to the public good will decline in this case.

5. Summary & Discussion

This paper extends the strategic delegation literature to solve for the optimal appointment decisions to a committee or legislature of members with heterogeneous preferences and a fixed budget to be divided between public and particularistic goods. It shows that strategic incentives matter, though not necessarily in ways previously explored in the literature. Districts face competing incentives over bargaining and political power. The former is typically realized by appointing pubs to bargain harder for contributions to the public good, while the latter is realized by appointing hogs who can be included in winning coalitions with proposers handing out pork projects. The paper shows that in this model, the choice for hog districts depends heavily on their preference for pork. Provided that the relative preference is not too large, they are willing to strategically delegate to pub legislators. Pub districts face the same competing incentives, but they always choose to appoint for bargaining power in equilibrium, even if that means appointing a hog legislator as in the boundary equilibria. Taken together this means strategic delegation works to increase public goods contributions. We show that equilibria exist where a majority of hog districts appoint a legislature comprised mostly of pub representatives. Comparative statics show that when strategic delegation occurs and there are more pub representatives than districts, a larger legislature will have a higher proportion of pub representatives and will allocate more funds to public goods.

Since this paper shares the same incentives for bargaining and political power as Harstad (2010), it is natural to ask how the results compare given the strict majority voting requirement used in the VW model. Harstad finds that these types of small majority requirements induce "progressive" delegation, while large requirements effect "conservative" delegation (sending representatives with preferences close to the status quo). Consider the following alternative interpretation of the baseline model in this paper. Instead of districts appointing representatives, suppose it is n groups that each send a representative to a decision-making body. We might say that the group who sends a representative that induces the production of more public good is "progressive" in the sense that it prefers more equality (in terms of payouts) for each group. Conversely, we could say a group that appoints a representative more likely to bring back particularistic goods is "conservative" in so far as it is less concerned with matters of equality between groups.[23] In that sense we have shown that with a strict (i.e., small) majority voting requirement, strategic delegation tends to incentivize "progressive" delegation as in Harstad.

Both the division of the budget and the equilibrium bargaining coalitions will change in the VW subgame with a larger majority voting requirement. An interesting trade-off

[23] I thank Massimo Morelli for pointing out this interpretation of the model.

emerges for the districts. As the majority requirement increases, a district's legislator's vote is need with higher probability to achieve bill passage. This tends to incentivize sincere, or relatively less "progressive", delegation. At the same time, however, the proposer's power is diminished because of the need to placate a larger majority, which means a hog district may still find it profitable to appoint a pub legislator. Investigating these opposing incentives may help further understand the implications of differing majority requirements on bargaining outcomes, delegation decisions, and whether strategic delegation leads to "progressive" or "conservative" representatives.

The strategic delegation model in this paper illustrates some of the key incentives facing voters as they solve the optimal appointment choice problem. Yet it lacks important features that could prove useful in future research. For instance, the model does not allow districts to appoint multiple legislators as is the case for the U.S. Senate.[24] In addition, utility is assumed to be linear so that marginal utilities remain constant. If nonlinear utility was allowed, it would likely reduce the incentives for hogs to strategically delegate but not eliminate the incentive altogether. It might also create an incentive for pubs to strategically delegate if the probability of being allocated pork is high.

Finally, the model assumes only two types of voters and representatives. An additional extension to the current analysis would be to allow each district its own k_i. This makes calculating the equilibrium to the underlying bargaining model much more complicated as coalitions can become more complex. Even so, this would be interesting at both the level of the legislative subgame and at the level of delegation. For instance, given that multiple legislators have $k_i > 1$, who does a proposer in need of a particularistic vote approach? While one obtains more "bang-for-the-buck" from a legislator with a high k_i, this legislator also has the greatest incentive to hold out for an expensive pork project. Extending the model in this way would also make the delegation game more interesting by opening up the choice set to more than two options. It is an open question as to how this would affect incentives for both bargaining and political power.

[24]The delegation decision is not hard to solve when voters appoint multiple representatives in staggered elections and the types of the sitting representatives (both for the district and for other districts) are taken as given. It turns out that the only time a voter's choice is affected is when the sitting representative for her district is a hog. In this case, the median voter can increase the amount of pork her representative can attain as proposer by appointing an additional hog. This makes both hog and pub districts quicker to delegate to hogs than in the case of one representative. However, the more interesting problem is to solve a dynamic game which would show how the sitting representative's type was determined. This is left for future research.

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Appendix A. Correction to Volden & Wiseman

The results in the diverse legislature extension reported in Volden and Wiseman (2007) and modified in Volden and Wiseman (2008) do not properly capture equilibrium strategies in the bargaining game when preferences for particularistic goods (pork) are strong among particularistic-leaning (hog) legislators, and these legislators comprise a majority in the legislature. The results of these papers show that for $k_H \in A_{2c}$ no pure strategy equilibrium exists. Instead, Volden and Wiseman (2008) reports that a mixed strategy equilibrium exists in which a recognized collective-leaning (pub) legislator offers to fully fund the public good, and a recognized hog player mixes between offering a particularistic proposal with probability β and a mixed minimum-winning coalition proposal (mixed-MWC) with probability $1 - \beta$.[25] For $k_H \in A_{2d}$ they assert that a pure strategy equilibrium exists in which a recognized pub proposer contributes a large amount to the public good and then must "grease the wheels" with pork for enough hog players to form a majority. A recognized hog player, by contrast, offers a particularistic proposal.[26]

Intuitively, the problem with this proposed equilibrium is that for $k_H \in A_{2d}$ the pure strategy played by the hog proposer ensures the pub legislators a low continuation value: with probability $(n-m)/n$ the pub players receive no payoff in the next stage of the game. Regardless of how large k_H is—and thus how little relative value the public good has for the hog player—a recognized hog proposer can always exploit the pub's low continuation value in the proposed pure strategy equilibrium by deviating to offer a mixed-MWC proposal, which contains some level of public goods contribution. And yet, the mixed-MWC proposal also can not be the basis of a pure strategy equilibrium for a hog proposer because pork has a high relative value in this region, and a hog proposer could profitably deviate with a particularistic proposal.

As a result, the only equilibria that exist for $k_H > \frac{n+1}{2\delta}$ will involve mixed strategies by the hog proposer. These strategies are given in the following proposition. Define

$$\varphi = \frac{k_H - \frac{n+1}{2}}{k_H - \frac{n+1}{2\delta}}.$$

Note that $\varphi \geq 1$ by assumption.

Proposition 7. *Suppose $m \leq \frac{n-1}{2}$ and $k_H > \frac{n+1}{2\delta}$. The bargaining game has the following unique mixed-strategy equilibrium:*

[25] The mixing probability is $\beta = 1 - \frac{(n+1)/2}{\delta(\alpha_H/q)}$.

[26] Volden and Wiseman (2008) claims that the aforementioned mixed strategy equilibrium also exists when $k_H \in A_{2d}$.

(a) *If* $k_H \leq \frac{n-\frac{\delta m}{\varphi}}{\delta}$ *a pub proposer offers a collective proposal* ($y = 1$) *and it is approved unanimously. A hog proposer offers a particularistic proposal with probability* $\beta = 1 - \frac{(n+1)/2}{\delta k_H}$ *and with probability* $1 - \beta$ *a mixed-MWC proposal. Either proposal is approved by the corresponding minimum-winning coalition.*

(b) *If* $k_H > \frac{n-\frac{\delta m}{\varphi}}{\delta}$ *a pub proposer forms a winning coalition by allocating particularistic goods to* $\frac{n+1}{2} - m$ *hogs and putting the remainder towards the public good. A hog proposer offers a particularistic proposal with probability*

$$\lambda = \left(\frac{\frac{n-1}{2}}{n-m}\right)\left(1 - \frac{m/\delta}{k_H - (\frac{n+1}{2} - m)}\right),$$

and a mixed-MWC proposal with probability $1 - \lambda$. *Each proposal receives minimum-winning support.*

The proof follows the logic expressed in Volden and Wiseman (2008).[27] Uniqueness follows from the fact that the only other potentially profitable mixed strategies for a hog proposer must include a mixed-universal proposal. However, a proposer will not be indifferent between either a particularistic or mixed-MWC proposal and a mixed-universal proposal in this range for k_H since the latter requires a large contribution to the public good and leaves little pork for the proposer.

Appendix B. Proof of Proposition 1

A district making its delegation decision takes as given k_H and ω and best-responds to the actions of other districts. Let us denote $m' = m+1$ as the result of adding one additional pub to the legislature. In addition, primes (') will be added to indicate the payout should an additional pub be appointed. For example, $y_1^{H'} = y_1^H(m+1)$. The equilibrium contributions to pork and the public good for each case in the underlying bargaining model can be found in Volden and Wiseman (2007). We refer the reader to their paper for the equations and the theory behind the derivations.

The first four lemmas presented establish the conditions for which strategic delegation is optimal in the interior of the four regions represented in the intervals for k_H in Proposition 1 (A_1, A_{2a}, A_{2b}, A_{2c}, and A_{2d}). The final two lemmas consider strategic delegation in common border cases. We will refer back to these when showing the equilibrium within each interval for k_H.

[27] The proof is available by request.

Lemma 2. *Suppose the majority of legislators are pubs, $m \geq (n+1)/2$. A hog district will appoint a pub if*

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H - 1}{k_H}\right). \tag{B.1}$$

A pub district will always appoint sincerely.

Proof. Clearly, an L district has no incentive to delegate strategically. Doing so only increases the chances a hog is chosen the proposer, which necessarily decreases the expected amount contributed to the public good. Moreover, conditional on a hog being chosen proposer, the district is strictly worse off; if the proposer is its legislator, she funds some pork barrel spending (money the district would rather see put towards the public good), and if she is not the proposer, this move actually lowers the district's utility through a smaller y_{2a}^H. An H district, on the other hand, will appoint a pub if the expected value to doing so is greater than the expected value from delegating sincerely to a hog:

$$\left(\frac{m'}{n}\right) + \left(\frac{n-m'}{n}\right) y_1^{H'} > \left(\frac{m}{n}\right) + \frac{k_H \hat{x}_1}{n} + \left(\frac{n-m}{n}\right) y_1^H.$$

Substituting in $y_1^H = (\delta m)/(n(1-\delta) + \delta m)$, $y_1^{H'} = (\delta m')/(n(1-\delta) + \delta m')$, $\hat{x}_1 = (n(1-\delta))/(n(1-\delta) + \delta m)$, $m' = m + 1$, and rearranging yields the inequality above. □

Lemma 3. *Let $k_H \leq (n+1)/2 - m$, and suppose the majority of legislators are hogs, $m \leq (n-1)/2$. A hog district will appoint a pub if*

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H - 1}{k_H}\right) - \left(\frac{k_H}{n}\right)\left(\frac{k_H - 1}{k_H}\right). \tag{B.2}$$

A pub district will always appoint sincerely.

Proof. Here again the pub district has no incentive to appoint strategically. The reasons are the same as those discussed in Lemma 2. A hog district, on the other hand, will appoint a pub so long as the expected utility is greater than that from appointing a hog:

$$\left(\frac{m'}{n}\right) + \left(\frac{n-m'}{n}\right) y_{2a}^{H'} > \left(\frac{m}{n}\right) + \frac{k_H \hat{x}_{2a}}{n} + \left(\frac{n-m}{n}\right) y_{2a}^H.$$

Substituting $\hat{x}_{2a} = [n(1-\delta)]/[\delta(k_H+m)+n(1-\delta)]$, $y_{2a}^H = [\delta(k_H+m)]/[\delta(k_H+m)+n(1-\delta)]$, and the corresponding value for $y_{2a}^{H'}$ yields the inequality above. □

Lemma 4. *Let $(n+1)/2 - m < k_H \leq (n+1)/(2\delta)$, and suppose $m \leq (n-1)/2$. A hog district will appoint a pub if*

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H - 1}{k_H}\right). \tag{B.3}$$

A pub district will always appoint sincerely.

Proof. A pub district may now be willing to appoint a hog given that hogs allocate pork to $(n-1)/2 - m$ other hog legislators in the equilibrium of the subgame. Either type of district will appoint a pub if the expected value of doing so is greater than the expected value of appointing a hog:

$$\left(\frac{m'}{n}\right) + \left(\frac{n-m'}{n}\right) y_{2b}^{H'}$$
$$> \left(\frac{m}{n}\right) + \left(\frac{n-m}{n}\right)\left\{ y_{2b}^{H} + \frac{\theta \hat{x}_{2b}}{n-m} + \left(\frac{n-m-1}{n-m}\right)\left(\frac{(n-1)/2-m}{n-m-1}\right)\theta x_{2b}^{H}\right\}.$$

Substituting in $\hat{x}_{2b} = [(1-\delta)(2n - \delta n + 2\delta m + \delta)]/[2(n(1-\delta)+\delta m)]$, $x_{2b}^{H} = \delta(1-\delta)/[n(1-\delta)+\delta m]$, $y_{2b}^{H} = \delta m/[n(1-\delta)+\delta m]$, and the corresponding value for $y_{2b}^{H'}$ yields,

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{\theta-1}{\theta}\right).$$

The righthand side of this equation is only less than 1 if $\theta > 1$. Hence, a hog will appoint strategically if this condition holds. Pubs will never appoint strategically. □

Lemma 5. *Let* $\frac{n+1}{2\delta} \le k_H < \frac{n-\delta m/\varphi}{\delta}$, *and suppose* $m \le (n-1)/2$.[28] *A hog district will appoint a pub if*

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H}{\frac{n+1}{2\delta}}\right)\left(\frac{k_H-1}{k_H}\right). \tag{B.4}$$

A pub district will always appoint sincerely.

Proof. Either type of district will appoint a pub if the expected value of doing so is greater than the expected value of appointing a hog,

$$\left(\frac{m'}{n}\right) + \left(\frac{n-m'}{n}\right)(1-\beta) y_{2c}^{H'}$$
$$> \left(\frac{m}{n}\right) + \left(\frac{n-m}{n}\right)(\beta)\left\{\frac{\theta \hat{x}_{2c,1}}{n-m} + \left(\frac{n-m-1}{n-m}\right)\left(\frac{(n-1)/2}{n-m-1}\right)\theta x_{2c,1}^{H}\right\}$$
$$+ \left(\frac{n-m}{n}\right)(1-\beta)\left\{ y_{2c,2}^{H} + \frac{\theta \hat{x}_{2c,2}}{n-m} + \left(\frac{n-m-1}{n-m}\right)\left(\frac{(n-1)/2-m}{n-m-1}\right)\theta x_{2c,2}^{H}\right\},$$

where β is the probability a hog proposer offers a particularistic proposal. Substituting in

[28] $m = \frac{n-\delta m/\varphi}{\delta}$ defines the boundary between A_{2c} and A_{2d}. See Appendix A for the functional form of φ.

the functional forms of the above terms yields,

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H}{\frac{n+1}{2\delta}}\right)\left(\frac{\theta-1}{\theta}\right).$$

Notice again that the righthand side of the equation is only less than 1 if $\theta > 1$ implying pubs will not appoint strategically. Hogs appoint strategically if this condition holds.

The maximization problem is similar for $k_H > \frac{n-\delta m/\varphi}{\delta}$ only it must be changed to incorporate the pub's equilibrium strategy of greasing the wheels with pork. The threshold and strategic delegation results are unchanged. □

Lemma 6. *Fix k_H and suppose m satisfies values such that $(k_H, m) \in A_{2a}$ and $(k_H, m+1) \in A_{2b}$. A pub district prefers to strategically delegate to a hog if*

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_L}{k_L - k_H}\right)\left(\frac{k_L - 1}{k_L}\right). \tag{B.5}$$

A hog district always prefers to delegate sincerely.

Proof. Hogs will always delegate sincerely. A hog prefers sincere delegation because (i) she receives the chance at pork if she appoints a hog, and (ii) the amount contributed to the public good in A_{2a} with m pubs is greater than the amount contributed in A_{2b} with $m+1$ pubs.[29]

A pub district will appoint strategically if the expected payoff to doing so is greater than the expected payoff from appointing sincerely. The result of this calculation is the inequality above. One particular result we will need for part 1(b) of Proposition 1 is for the case of $m = \frac{n-1}{2} - 1$. A pub district prefers this outcome in A_{2a} to $(k_H, (n-1)/2) \in A_{2b}$ if the inequality in the lemma holds for this value of m,

$$k_L > \frac{k_H \delta\left(\frac{n+1}{2}\right) - n}{\delta\left(\frac{n+1}{2}\right) - n}. \tag{B.6}$$

□

Lemma 7. *Fix k_H and suppose m satisfies values such that $(k_H, m) \in A_{2b}$ and $(k_H, m+1) \in$*

[29] This is not obvious and must be shown. It is derived from the fact that in A_{2a} a hog proposer uses the public good to get all other legislators' support, both hogs and pubs. In A_{2b} the hog proposer contributes enough to the public good to get only the pubs' support and fills out her coalition with pork for enough hogs to get a majority. It is easy to show that $y_{2a}^{H} > y_{2b}^{H'}$.

A_1. A hog district prefers to strategically delegate to a pub if

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{k_H - 1}{k_H}\right). \tag{B.7}$$

A pub district always prefers to delegate sincerely.

Proof. A district will prefer to delegate to a hog if the expected payoff to doing so is greater than the expected payoff to appointing a pub. We suppress the mathematics here, but the result is,

$$\frac{m+1}{n} < 1 - \left(\frac{1}{\delta}\right)\left(\frac{\theta - 1}{\theta}\right).$$

The righthand side is only less than one if $\theta > 1$. Hence, pubs will delegate sincerely and hogs will delegate strategically if this equation is satisfied for $\theta = k_H$. $\square$

To calculate the equilibrium we must consider each possible value of m and determine whether it can be an equilibrium for a given ω. In formulating the equilibrium it is necessary to identify which type(s) are playing strategically so as to check that the actions are individually rational for strategic and sincere players. This is made easier by introducing the following notation: let $\langle \theta, d \rangle_i$ denote district i's type and choice of representative.

We present here the proof of the equilibrium for part 1 of Proposition 1 where $k_H \leq \frac{n}{n - \delta(n-1)/2}$. The proofs for parts 2 and 3 and Proposition 2 follow similar logic and also make use the above lemmas.[30] The remainder of the proof is available by request.

Consider $k_H \leq \frac{n}{n - \delta(n-1)/2}$. We consider six possible values for m but begin first with an observation.

Remark 2. *For $k_H \leq \frac{n}{n - \delta(n-1)/2}$ the function $m = n\left(1 - \frac{k_H - 1}{\delta k_H}\right) - 1$ defined implicitly by Lemma 2 is greater than or equal to $(n+1)/2$.*

1. No equilibrium exists such that $m < \frac{n-1}{2} - 1$. Suppose not. Using our observation combined with Lemma 3 it is clear that for m in this range, a district characterized by
   (a) $\langle L, H \rangle$ will deviate and appoint an L,
   (b) $\langle L, L \rangle$ will not deviate,
   (c) $\langle H, L \rangle$ will not deviate, and
   (d) $\langle H, H \rangle$ will deviate and appoint an L.

[30] The other regions also require the use of a pair of lemmas dealing with border cases not presented here.

Any equilibrium must contain districts not willing to deviate, $\langle L, L\rangle$ and $\langle H, L\rangle$, but this implies $m = n$, a contradiction.

2. $m = \frac{n-1}{2} - 1$ can be an equilibrium depending on ω, k_H, and k_L. Given m takes this value, and using the remark, Lemma 3 and Lemma 6, a district characterized by
   (a) $\langle L, H\rangle$ will not deviate if k_L satisfies equation (B.6),
   (b) $\langle L, L\rangle$ will not deviate,
   (c) $\langle H, L\rangle$ will not deviate, and
   (d) $\langle H, H\rangle$ will not deviate.

   These conditions imply that an equilibrium always exists when $\omega \leq \frac{n-1}{2} - 1$ and exists for any ω only if k_L satisfies equation (B.6).
3. $m = \frac{n-1}{2}$ can not be an equilibrium. Using the remark combined with Lemmas 6 and 7, a district characterized by
   (a) $\langle L, H\rangle$ will deviate and appoint an L,
   (b) $\langle L, L\rangle$ will deviate and appoint an H if k_L satisfies equation (B.6),
   (c) $\langle H, L\rangle$ will deviate and appoint an H, and
   (d) $\langle H, H\rangle$ will deviate and appoint an L.

   Hence, even if k_L does not satisfy equation (B.6), no equilibrium will exist for any ω.
4. No equilibrium exists such that $\frac{n+1}{2} \leq m < n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1$. Suppose not. Using the remark and Lemmas 2 and 7, then for m in this range a district characterized by
   (a) $\langle L, H\rangle$ will deviate and appoint an L,
   (b) $\langle L, L\rangle$ will not deviate,
   (c) $\langle H, L\rangle$ will not deviate, and
   (d) $\langle H, H\rangle$ will deviate and appoint an L.

   Any equilibrium must contain districts not willing to deviate, $\langle L, L\rangle$ and $\langle H, L\rangle$, but this implies $m = n$, a contradiction.
5. $m = n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1$ will be an equilibrium in which all L districts appoint sincerely for any $\omega \leq n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1$. Using the remark and Lemmas 2 and 7, a district characterized by
   (a) $\langle L, H\rangle$ will deviate and appoint an L,
   (b) $\langle L, L\rangle$ will not deviate,
   (c) $\langle H, L\rangle$ will not deviate, and
   (d) $\langle H, H\rangle$ will not deviate.

   This implies that this will be an equilibrium so long as all pub districts appoint sincerely. Thus if $\omega \leq n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1$ this will be an equilibrium.

6. Finally, $m = \omega > n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1)$ is an equilibrium provided both H and L districts appoint sincerely. Using the remark and Lemma 2, a district characterized by

   (a) $\langle L, H\rangle$ will deviate and appoint an L,
   (b) $\langle L, L\rangle$ will not deviate,
   (c) $\langle H, L\rangle$ will deviate and appoint an H, and
   (d) $\langle H, H\rangle$ will not deviate.

   Hence, $m = \omega$ for $\omega > n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1$ is an equilibrium if and only if all districts appoint sincerely.

Appendix C. Proof of Propositions 5 and 6

The comparative statics are given for each non-boundary equilibrium value of m^* shown in Propositions 1 and 2 (the comparative statics for parts 1(b) and 2(c) of Proposition 1 and parts 1(b) and 2(b) of Proposition 2 are straightforward to calculate). Clearly, if $m^* = \omega$ the equilibrium number of pubs does not depend on δ, k_H, or n.

(1) Suppose $k_H \leq \frac{n}{n-\delta\left(\frac{n-1}{2}\right)}$ and $m^* = n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - 1 > 0$.

$$\begin{aligned}
\frac{\partial(m^*/n)}{\partial\delta} &= \frac{\left(\frac{k_H-1}{k_H}\right)}{\delta^2} > 0 \\
\frac{\partial(m^*/n)}{\partial k_H} &= -\left(\frac{1}{\delta}\right)\left(\frac{1}{k_H^2}\right) < 0 \\
\frac{\partial(m^*/n)}{\partial n} &= \left(\frac{1}{n^2}\right) > 0.
\end{aligned}$$

(2) Suppose $\frac{n}{n-\delta\left(\frac{n-1}{2}\right)} < k_H \leq \frac{n+1}{2\delta}$ and $m^* = n\left(1 - \dfrac{k_H - 1}{\delta k_H}\right) - k_H > 0$.

$$\begin{aligned}
\frac{\partial(m^*/n)}{\partial\delta} &= \frac{\left(\frac{k_H-1}{k_H}\right)}{\delta^2} > 0 \\
\frac{\partial(m^*/n)}{\partial k_H} &= -\left(\frac{1}{\delta}\right)\left(\frac{1}{k_H^2}\right) - \frac{1}{n} < 0 \\
\frac{\partial(m^*/n)}{\partial n} &= \left(\frac{k_H}{n^2}\right) > 0.
\end{aligned}$$

The comparative statics for the other equilibrium value of m^* are the same as in (1).

(3) Suppose $k_H > \frac{n+1}{2\delta}$ and $m^* = n\left(1 - \frac{k_H - 1}{\delta k_H}\left(\frac{k_H}{\frac{n+1}{2\delta}}\right)\right) - 1 > 0$. Note that $m^* > 0$ if and only if $(k_H - 1) < (n+1)/2$.

$$\begin{aligned}
\frac{\partial(m^*/n)}{\partial\delta} &= 0 \\
\frac{\partial(m^*/n)}{\partial k_H} &= -\left(\frac{1}{\frac{n+1}{2}}\right) < 0 \\
\frac{\partial(m^*/n)}{\partial n} &= \frac{1}{n^2} + \frac{k_H - 1}{2\left(\frac{n+1}{2}\right)^2} > 0.
\end{aligned}$$