Abstract

Let $C[0,1]$ be the space of all continuous real-valued functions defined on $[0,1]$ with the supremum norm. There are some natural operations in $C[0,1]$ as addition, multiplication, minimum and maximum, studied in [BWW, WA]. All of them are continuous but only addition, minimum and maximum are open as mappings from $C[0,1] \times C[0,1]$ to $C[0,1]$.

(Fremlin’s example) [BWW] In 2004, D. H. Fremlin observed that for $f: [0,1] \to \mathbb{R}$, $f(x) = x - \frac{1}{2}$, one has $f^2 \not\in B^2(f, \frac{1}{2}) \setminus \int B^2(f, \frac{1}{2})$. Hence multiplication is not an open mapping from $C[0,1] \times C[0,1]$ into $C[0,1]$.

In [BWW] it is shown that the multiplication in $C[0,1]$ is a weakly open operation i.e. image of every non-empty open set has a non-empty interior. This was generalized in [KO] for $C(0,1)$ and in [BM] for $C(X)$, where $X$ is an arbitrary interval.

We study problem, stated in [BSW] Question 18.24 and 18.25], of openness and weak openness in the space $BV[0,1]$ of functions of bounded variation and in the space $CBV[0,1]$ of continuous functions of bounded variation, both defined on $[0,1]$, with the norm $\|f\|_{BV} = |f(0)| + V^1_0(f)$.

We shall show that multiplication is an open operation in $(BV[0,1], \|\|_{BV})$ and weakly open in $(CBV[0,1], \|\|_{BV})$. Moreover, we show that multiplication is not uniformly open and we thereby solve in the negative the problem of whether open bilinear maps are automatically uniformly open, stated in [BBS]. Furthermore, we will study addition and multiplication in $CBV[0,1]$ with the Adams metric $\varrho_A(f,g) = \int_0^1 |f(x) - g(x)|dx + |V^1_0(f) - V^1_0(g)|$.

Presented results are obtained jointly with Małgorzata Turowska.
Bibliography


